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Numerical Analysis and Computation of Nonlinear Partial Differential Equations.

Final Report

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Work which involved both error estimates and computations on a time-dependent problem modeling the deformations of a viscoelastic solid was completed during the term of this report. Also two algorithms were derived and implemented for the KdV equation with wavemaker boundary conditions.

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FINAL REPORT

Significant progress was made on the time-dependent problem involving the equation

$$u_{tt} - \Delta u_t - \nabla \cdot \left(\frac{\omega'(|\nabla u|)}{(|\nabla u|} \nabla u\right) = f \text{ in } \Omega \subset R^2$$

which models antiplane shear deformations of certain viscoelastic solids. The nonlinear function ω is a double well potential.

We derived an energy preserving implicit time discretization scheme and implemented it in two space dimensions to study numerically the long time behavior of the solution. This scheme was analyzed for the one-dimensional problem with Soren Jensen.[1] Since this was our first computation with this problem we kept the method as simple as possible; fixed point iteration was used to solve the nonlienar systems and preconditioned conjugate gradients for the linear system. Previously, optimal order error estimates were derived in collaboration with Lars B. Wahlbin of Cornell for the semi-discrete scheme (Discretized in space by finite elements). Using our code we verified these error estimates. We also computed solutions in the case where ω was a double well potential and where it was the convexification of the well. In the convexified case u tended to a steady state pattern that was identical to the one found in Goodman, Kohn, and Reyna[2] who computed solutions in the static case. In the nonconvex situation u tended to nearly the same solution. We plan to write up both the theoretical and numerical results and submit them for publication soon.

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[1] Behaviour in the large of numerical solutions to one-dimensional nonlinear viscoelasticity by continuous time Galerkin methods (Accepted for publication by *Comp. Meth. Appl. Mech. Eng.*)

[2] Numerical Study of a relaxed variational problem from optimal design, *Comp. Meth. Appl. Mech. Eng.*, 57(1986) 107-127.

We began testing the continuous time Galerkin (CTG) method by implementing two numerical schemes on the generalized Kortewig-de Vries (KDV) equation

$$u_t + f(u)_x + \varepsilon u_{xxx} = 0$$

where, typically, $f(u) = u^5$ or $f(u) = u^7$. Instead of the usual periodic boundary conditions (BC) we used wavemaker BC[3] as suggested to us by Jerry Bona of Penn State. We intend to use our codes to study the behavior of the solutions. We will begin by investigating the stability of solitary waves.

The numerical schemes we are using are described in French and Schaeffer[4]. One is very similar to the Crank-Nicolson method and would conserve the L^2 norm of the solution in the periodic case. The other is based on a splitting of the equation suggested by Winther[5] and will conserve the third invariant in the periodic case.

Another extension of our work on CTG methods is in the area of space-time finite element methods.[6,7] Here one uses finite elements to discretize a domain that includes the time dimension rather than using finite differences in time and some other method is space as is usually done. The key advantage of the space-time approach is it allows unstructured meshes in time which permits efficient mesh refinement. A disadvantage of these schemes is they

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[3] J. Bona and R. Winther, The Kortewig-de Vries equation in the quarter plane, continuous dependence results, *Differential and Integral Equations*, 2(1989) 228-250.

[4] Continuous finite element methods which preserve energy properties nonlinear problems (Accepted for publication by *Appl. Math. Comp.*)

[5] A conservative finite element method for the Kortewig-de Vries equation, *Math Comp.* **34**(1980) 23-43.

[6] T.J.R. Hughes and E.M. Hilbert, Space-time finite element methods for elastochynamics: formulations and error estimates, *Comp. Meth. Appl. Mech. Eng.*, 66(1988) 339-363.

[7] C. Johnson, *Numerical Solution of Partial Differential Equations by the Finite Element Method*, Cambridge University Press, (Section 9.9).

are less stable - at least theoretically - and usually require the introduction of certain terms that enhance stability without compromising accuracy. Thus the derivation of algorithms can be difficult. Currently we are attempting to derive space-time schemes for dispersive wave equations such as the KdV and Schrödinger equations. We plan to write a computer program that will track a single soliton solution accurately and efficiently.